It is often said that the role of a successful teacher is one of "lighting the fire" of curiosity in students rather than "filling the cup" of knowledge. I believe that both roles are essential for a college educator. Students should get what they pay for, and my immediate goal as a teacher is to help them absorb as much of the relevant course material as possible. Setting clear course expectations, being organized and well-prepared, and providing ample feedback to my students are of fundamental importance to me. To these ends, an up-to-date course website (usually via a learning management system), consistent office hours (regularly scheduled and by appointment), plenty of supplemental material and exercises, and online access to detailed solutions to homework and exam problems are just a few of the resources that I always provide during my courses. On the other hand, I believe that my role as a "fire starter" is to help students discover how to think mathematically, which I do by dispelling myths about mathematics, fostering students' ownership and accountability in the learning process, and being flexible in my methodology.

One of the most pervasive myths about math is that it is all about rote memorization and application of formulas, which I think is a product of the bottom-up approach to pedagogy that is followed in most primary and secondary schools, and even in some post-secondary institutions. We take mathematical truths discovered painstakingly by trial-and-error over centuries, silo the information and organize it linearly, and then present it as something static and immutable. Though this approach has merits (the information has to be organized somehow!), it can obscure connections between seemingly unrelated topics, and, worse, hide that the process of doing math is a process of failing repeatedly. It is no wonder that so many students develop such a fear of math during their pre-college experience! One technique that I use to dispel this myth is to teach examples before theory, because I believe that we should teach math by using something small to hint at something big, building students' understanding like a blurry painting which becomes more and more clear over time. When presenting a new topic, I like to motivate it with an example and allow the students to first attempt to tackle the problem with the approaches that they already know, so that they can discover on their own that something new or more general is needed. I make it a point not to tell them right away that I have set them up to "fail." When the students begin to make the connection that something new is necessary, I can present the relevant theorem, knowing that they have a good example in the back of their minds to accompany the explanation. If I am lucky, I can even use that same example to give a hint of the proof of the theorem. This parallels what mathematical discovery is really like, and I also think that it prepares students to think critically and come up with creative solutions to problems they will encounter in any job after higher education, all while fostering a growth mindset. Another technique that I like to employ to help students see the interconnectedness of seemingly unrelated mathematical topics is to present the students with "math riddles" which hint at beautiful ideas directly or indirectly related to the course material. For example, prior to introducing the natural log function in calculus, I might ask students how to detect which one of 1000 bottles of grape juice is poisoned using only 10 thirsty rats to remind them of logarithms. Or, I might ask them to determine whether e^π or π^e is larger without using a calculator. This illustrates the power of logarithmic differentiation and also opens up a discussion into what it means to have an irrational exponent in an expression.

A focus on examples before theory also spurs students to ask questions and make connections so that they take more ownership in their learning. I took this philosophy to the extreme in Spring of 2017, when I ran a discussion section as a "math lab," where the students worked in groups on worksheet prepared by the professor that I TA'ed for and me to augment their learning in the Calculus II lecture. The worksheets were designed to walk the students through multi-part problems, more challenging than typical homework problems, which built on material from the last lecture and hinted at new material in the next lecture. While they worked, I would visit each group in turn to engage with the ideas they came up with and give them hints when they were stuck. This project-based learning approach to the discussion was incredibly well-received, and it was rewarding to see what new connections and questions the students could come up with all on their own. For example, one student was curious to find a series made out of elementary functions for which the methods we learned in class could not determine easily whether it converged or diverged, and successfully found one online! Encouraging students to open their textbooks is something else that I emphasize in an attempt to foster students' self-reliance. I assign textbook readings as an integral part of any homework assignment, and will sometimes give quizzes on the reading material. When students ask me questions in office hours, I will often encourage them to find a similar example in the textbook and emulate the technique. Sometimes, I will encourage them to search for a hint online and help them parse it as necessary. Another technique I have used to emphasize students' accountability is to do "exam breakdowns" after an exam that didn't go as well as expected. This entails having the students make a list of factors that led to them not performing the way that they wanted, and then organizing them into things that were in their control as students, my control as the teacher, and out of anyone's control. Then we make a deal as a class to address the factors over which we have control. This exercise helps the students to see that I am on their side, while emphasizing the role they have of reinforcing their own understanding of course material inside and outside the classroom. In an age where a college education is becoming more of a parental expectation than a conscious decision to invest in one's own future, I think such attitudes are especially important to develop in students. And it goes almost without saying that part of thinking mathematically is having an internal locus of control.

I am wary of saying that there is a single best methodology to follow when running a class. Everyone learns differently, and a class which favors more discussion and group work may respond poorly to the traditional approach of a professor lecturing for an hour while students sit quietly, taking notes and working examples on their own. On the other hand, some quieter classes prefer the latter approach, feeling that too much discussion in a classroom is a distraction to covering more material. Thus, I think it is important as a teacher to be flexible in one's preferred method of engaging students, being fluent both in the art of lecturing clearly and stimulating discussion when appropriate. I had one stand-out experience with this extreme in Summer of 2016, when I taught a class called Paradoxes and Infinities to gifted high school students as part of John's Hopkins University's Center for Talented Youth Program, during which I was encouraged to adopt a variety of inquiry-based methods, group discussions, and exploratory, hands-on activities, to which the students responded quite positively. In the future, I know that many of my students will need to rely on technology to solve problems, so it is especially important to me to foster technological literacy. In fact, technology has opened up a treasure trove of possibilities for engaging meaningfully with students and has become a core aspect of my methodology. In the last few classes I have taught, I write lecture notes on my tablet and project them at the front of the room. This makes it easier to use a variety of colors to make notes and drawings more clear. At the end of the class I can export my notes to a pdf and put them online so that students have easy access to them. I also use the TI-83, MATHEMATICA, WolframAlpha, Sage, and other online tools to give students visual demonstrations of course topics such as Taylor series that would not have been possible several decades ago. One of my favorite exercises to do to motivate power series is to use Mathematica to plug small powers of 10 (e.g. 10^{-6}) into the function $f(x) = \frac{x}{1-x-x^2}$ and ask the students why the Fibonacci numbers appear in the decimal output. Such demonstrations inspire curiosity, and I believe that being flexible about how information is presented is indispensable for stimulating mathematical thinking.